RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College under University of Calcutta)

B.A./B.Sc. FIRST SEMESTER EXAMINATION, JANUARY 2015

FIRST YEAR

Date : 10/01/2015 **STATISTICS (General)**

Time : 11 am – 1 pm **Paper :** I Full Marks : 50

[Use a separate Answer Book for each group]

Group – A

(*Answer any three questions from Q.No. 1 - 6*) [3×5]

1. What do you mean by dispersion? Name a few measures of dispersion. Show that standard deviation does not depend on the change of origin but it is dependent on the change of scale. [1+1+3]

2. Show that for a set of 'n' positive values of a variate : $AM \geq GM \geq HM$.

3. Let 's' and R be, respectively, the standard deviation and range of a set of n values of X. Show that :
$$\frac{R^2}{2n} \leq s^2 \leq \frac{R^2}{4}$$

4. Show that Mean absolute deviation is minimum when the deviations are taken from median.

5. Show that $\beta_2 - \beta_1 - 1 \geq 0$ where the symbols have usual meanings.

6. Two regression equations are given as: 2X + 3Y = 5; 3X + 4Y = 7. What are the values of $\bar{X}$ and $\bar{Y}$?
The correlation between two variables (X & Y) is zero. Are X and Y independent? [2×2·5]

(*Answer either Q.No. 7 or 8*) [1×10]

7. Define Pearson's product moment correlation coefficient ρ_{xy} and discuss its limitations.

   Show that $-1 \leq \rho_{xy} \leq 1$

   Show how ρ_{xy} changes for change of origin and scale.

8. Derive the least square linear regression equations of : y or x and x on y.
   Also find the angle between the two regression lines.
   Let X and Y be independent variables with standard deviations σ_x and σ_y. Show that the correlation coefficient between X and X + Y is $\sigma_x \Big/ \sqrt{\sigma_x^2 + \sigma_y^2}$.

Group – B

(*Answer any three questions from Q.No. 9 - 14*) [3×5]

9. Consider an experiment of drawing two cards at random from a bag containing 4 cards marked with integers 1 through 4.
   a) Find the sample space S_1 of the experiment if the first card is replaced before the second is drawn.
   b) Find the sample space S_2 of the experiment if the first card is not replaced. [5]

10. A person X fails to remember the last digit of the telephone number of his house physician Dr. Y. What is the probability that at most 4 attempts are necessary to find the actual number? [5]

11. Five cards are drawn from a full pack of 52 cards. Find the probabilities that
    a) they are of different denominations
    b) 2 are of same denomination and 3 are of different denominations. [5]

12. Suppose an urn contains 'a' white balls & 'b' black balls. If you draw a ball & it happens to be white, what is the probability of a white ball in the second draw (without replacement)? If you do not know the result of the first draw, then find the probability of a white ball in the second draw (without replacement)? [0·5+4·5]

13. A man takes a step forward with probability 0·7 & backward with probability 0·3. Find the probability that at the end of 13 steps he is 3 steps forward from the starting point. [5]

14. A, B and C can solve a problem with probabilities $\frac{1}{4},\frac{1}{3},\frac{1}{6}$ respectively. If they all try, what is the probability that the problem will be solved. [5]

(*Answer either <u>Q.No. 15 or 16</u>*) [1×10]

15. a) Let us select five cards at random & without replacement from an ordinary deck of playing cards. Find the p.m.f. of the random variable X, the number of hearts in the five cards. Also determine $P[X \le 1]$. [3]

    b) Let F(x) be the distribution function of a random variable X. Prove that $P[a \le X < b] = F(b-0) - F(a-0)$. [2]

    c) The life in hours of a certain kind of ratio tube has the probability density function

$$f(x) = \begin{cases} 100/x^2 ; & x \ge 0 \\ 0 \quad ; & \text{elsewhere} \end{cases}$$

    What is the probability that none of three tubes in a given radio set will have to be replaced during the first 150 hours of operation? What is the probability that all three of the original tubes will have to be replaced during the first 150 hours? [5]

16. a) Let the subsets $C_1 = \left\{\frac{1}{4} < x < \frac{1}{2}\right\}$ and $C_2 = \left\{\frac{1}{2} \le x < 1\right\}$ of the space A = {x : 0 < x < 1} of the random variable X be such that $P_X(C_1) = \frac{1}{8}$ & $P_X(C_2) = \frac{1}{2}$. Find $P_X(C_1 \cup C_2)$, $P_X(C_1^C)$ and $P_X(C_1^C \cap C_2^C)$. [3]

    b) The distribution of a discrete random variable X is given by $P[X=-1] = P[X=0] = P[X=1] = 1/4$; $P[X=-2] = 1/16$, $P[X=2] = 3/16$ Find the median of X. [3]

    c) A lot of 12 television sets include 2 with white cords. If 3 of the sets are chosen at random for shipment, how many sets with white cords can the shipper expect to send? [2]

    d) Let the random variable X have p.m.f

$$f(x) = \begin{cases} p & \text{for} \quad x = -1,1 \\ (1-2p) & \text{for} \quad x = 0 \\ 0 & \text{elsewhere} \end{cases}$$

    where $0 < p < \frac{1}{2}$.

    Find the measure of Kurtosis as a function of p. [2]

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